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INFERENCE OF SIGNS OF INTERACTION EFFECTS IN SIMULTANEOUS GAMES WITH INCOMPLETE INFORMATION

BY URE O DE PAULA AND XUN TANG[1]

T . W - , , . T - . T ( ) (DGP). A - , DGP. W U S , -

, , (S (2009))
(B , H , K ,
N (2010)).

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BNE BNE
. I ,
. T -
,
, - -

ε - (CDF) $F_{\varepsilon|X}(\cdot|x)$. T i 1 $U_{1i}(X, \varepsilon_i) \equiv u_i(X) + (\sum_{j \neq i} D_j)\delta_i(X) - \varepsilon_i$, $U_{0i}(X, \varepsilon_i)$ 0. I , $u_i(X)$ 1 i. M $\delta_i(X)$ i' j 1. T $(u_i, \delta_i)_{i=1}^{N}$ $F_{\varepsilon|X}$. W $F_{\varepsilon|X}$ -

ASSUMPTION 1: $x \in \Omega_X$, $F_{\varepsilon|X}(\cdot|x) = \prod_{i \leq N} F_{\varepsilon_i|X}(\cdot|x)$ $\mathbb{R}^N$.

A 1 ε_i' $X = x$. I X - , . T . A i B $s_i : \Omega_{X, \varepsilon_i} \to \{0, 1\}$. L $S_i(X, \varepsilon_i)$ i,

$$S_i(X, \varepsilon_i) = \begin{cases} 1, & u_i(X) + \delta_i(X) \sum_{j \neq i} \mathbb{E}[S_j(X, \varepsilon_j)|X, \varepsilon_i] - \varepsilon_i \geq 0, \\ 0, & . \end{cases}$$

U A 1, $\mathbb{E}[S_j(X, \varepsilon_j)|X = x, \varepsilon_i] = \mathbb{E}[S_j(X, \varepsilon_j)|X = x] \equiv p_j(x)$, B N (BNE) (x) $p(x) \equiv [p_1(x), \ldots, p_N(x)]$ $x \in \Omega_X$,

$$p_i(x) = F_{\varepsilon_i|X=x}\Big(u_i(x) + \delta_i(x) \sum_{j \neq i} p_j(x)\Big) \quad i = 1, \ldots, N, \tag{1}$$

$p_i(x)$ i' 1 x $F_{\varepsilon_i|X}$ ε_i X. L $\mathcal{L}_{x,\theta}$ BNE (p (1)) x $\theta \equiv \{(u_i, \delta_i)_{i=1,\ldots,N}, F_{\varepsilon|X}\}$. T - BNE x B ' $F_{\varepsilon_i|X}$ A 1. I BNE, $F_{\varepsilon|X}$, u_i, δ_i.

T (. ., δ_{ij}). N , '

(S 3 S M (P. T (2012))). T , , ( ) , (., $f_i(x, D_{-i}) = $ $_{j \neq i}(D_j)$), (.,

PROOF: U A 1, D_i $\sum_{j\neq i} D_j$ -
x BNE p^l $\mathcal{L}_x$.

f *f* ( ). S BNE - -
; , $\mathcal{L}_x^+$, p^l . T $p_i^*(x) = p_i^l(x)$, $y_i^*(x) = \sum_{j\neq i} p_j^l(x)$,
$\tilde{y}_i^*(x) = p_i^l(x)\sum_{j\neq i} p_j^l(x)$ i x. H $\tilde{y}_i^*(x) = p_i^*(x)y_i^*(x)$
i.

f ( ). S $\mathcal{L}_{x,\theta}^+$ x. T
i $p^l, p^k \in \mathcal{L}_x^+$ $p_i^l \neq p_i^k$. A
i, $\delta_i(x)$. B ,

$$(3)\qquad \Delta_i(x) \equiv \tilde{y}_i^*(x) - p_i^*(x)y_i^*(x)$$

$$= \int_{p^l\in\mathcal{L}_x^+} p_i^l(x)y_i^l(x)\,d\Lambda_x - \int_{p^l\in\mathcal{L}_x^+} p_i^l(x)\,d\Lambda_x \int_{p^l\in\mathcal{L}_x^+} y_i^l(x)\,d\Lambda_x.$$

S $\delta_i(x) > 0$. T (1)
h_i $y_i^l(x) = h_i(p_i^l(x)) \equiv$
$(F_{\varepsilon_i|X}^{-1}(p_i^l(x)) - u_i(x))/\delta_i(x)$ p^l $\mathcal{L}_{x,\theta}$.[7] T x , (3)

$$\tilde{y}_i^*(x) - p_i^*(x)y_i^*(x)$$

$$= \int_0^1 h_i(z)z\,d\tilde{\Lambda}_{i,x}(z) - \int_0^1 z\,d\tilde{\Lambda}_{i,x}(z)\int_0^1 h_i(z)\,d\tilde{\Lambda}_{i,x}(z),$$

$z \equiv p_i^l(x)$ $\tilde{\Lambda}_{i,x}$ $p_i^l(x)$
Λ_x $\mathcal{L}_x$. T (3)
Z :

$$\begin{aligned}
(Z, h_i(Z)) &= \mathbb{E}\big[\big(Z - \mathbb{E}(Z)\big)\big(h_i(Z) - \mathbb{E}(h_i(Z))\big)\big]\\
&= \mathbb{E}\big[(Z - \mathbb{E}(Z))\big(h_i(Z) - h_i(\mathbb{E}(Z))\big)\big]\\
&\quad + \mathbb{E}\big[(Z - \mathbb{E}(Z))\big(h_i(\mathbb{E}(Z)) - \mathbb{E}(h_i(Z))\big)\big]\\
&= \mathbb{E}\big[(Z - \mathbb{E}(Z))\big(h_i(Z) - h_i(\mathbb{E}(Z))\big)\big].
\end{aligned}$$

B h_i [0, 1] x, $z_1 > z_2 \Rightarrow h_i(z_1) >$
$h_i(z_2)$. C , $(z - \mathbb{E}(Z))(h_i(z) - h_i(\mathbb{E}(Z))) > 0$ $z \neq \mathbb{E}(Z)$,
, $\tilde{\Lambda}_{i,x}$
$\mathcal{L}_x^+$. H $\tilde{y}_i^*(x) - p_i^*(x)y_i^*(x) > 0$ BNE
- x. T $\delta_i(x) < 0$

[7]T h_i θ x . W
.

T ... , ... (i, x) ... , ... (... , ... , ...). I ... , ... (i, x). W ...

... f $\delta_i(x)$

C ... $x \in \Omega_X$ ... i. T ... $(\delta_i(\cdot))$ ... i ... BNE ... DGP ... F_X. T ... , ... $\delta_i(x) > (<)\ 0$ ... $\tilde{y}_i^*(x) - p_i^*(x) y_i^*(x) > (<)\ 0$. F ... , ... $\delta_i(x) > (<)\ 0$ ... DGP ... x, ...

X $\tilde{X}_0$ - - . X - $(\tilde{X}_i)_{i\le N}$ $\tilde{X}_i$ - i . F , $\tilde{X}_i$ i. T - $(\varepsilon_i)_{i\le N}$ - ( ) . I ’ ( ) i’ $\tilde{X}_0$ $\tilde{X}_i$, $F_{\varepsilon_i|X} = F_{\varepsilon_i|X_i}$, $X_i = (\tilde{X}_0, \tilde{X}_i)$. F (i, x), $Y_i(x_i) \equiv \{x' : x'_i = x_i\}$ i x. W .

ASSUMPTION 2: i, *f* X (X_i) $u_i(x) = u_i(x_i)$, $\delta_i(x) = \delta_i(x_i)$, $F_{\varepsilon_i|X=x} = F_{\varepsilon_i|X_i=x_i}$ *f* x.

T $(\delta_i(x))$ (i BNE x) : ( ) P i BNE $u_j(x'), \delta_j(x'), F_{\varepsilon_j|X=x'}$ $Y_i(x_i)$. ( ) B , i (1) x' $Y_i(x_i)$. ( ) O ’ - i’ x' $Y_i(x_i)$ i, x' $Y_i(x_i)$ A 2. C , ’ , i

M C (S 5). W -
P 2. L g ,
$D_{i,g}$ i g. D

$$\Psi_i(x_i) \equiv \mathbb{E}\ D_{i,g} \underset{j\neq i}{\quad} D_{j,g}\ X_g \in Y_i(x_i)$$

$$-\mathbb{E}[D_{i,g}|X_g \in Y_i(x)]\mathbb{E} \underset{j\neq i}{\quad} D_{j,g}\ X_g \in Y_i(x_i)$$

PROPOSITION 2: 1 2 . ( ) x, $(\delta_i(x)) =$ $(\Psi_i(x_i))$ *f* i *f* $\Lambda^*_{x_i}$ i' ( ) $\Lambda^*_{x_i}$ i' *f* *f* $\Psi_i(x_i) \neq 0$.

PROOF: C (i,x) $\Lambda^*_{x_i}$. T (1) A 2 h_i $y^l_i(z) = h_i(p^l_i(z))$ $z \in Y_i(x_i)$ $p^l \in \mathcal{L}^+_x$, $h_i(\cdot) \equiv (F^{-1}_{\varepsilon_i|x}(\cdot) - u_i(x))/\delta_i(x)$. T h_i i' BNE $j \neq i$. I $\Lambda^*_{x_i}$ i' ,

$$\text{(5)}\qquad \Psi_i(x_i) = \underset{p\in\mathcal{L}^*_{x_i}}{\quad}\mathbb{E}\ D_i \underset{j\neq i}{\quad} D_j\quad p, X \in Y_i(x_i)\ \ d\Lambda^*_{x_i}$$

$$- \underset{p\in\mathcal{L}^*_{x_i}}{\quad}\mathbb{E}[D_i|p, X \in Y_i(x_i)]\, d\Lambda^*_{x_i}$$

$$\cdot \underset{p\in\mathcal{L}^*_{x_i}}{\quad}\mathbb{E} \underset{j\neq i}{\quad} D_j\ p, X \in Y_i(x_i)\ \ d\Lambda^*_{x_i}$$

$$= \underset{p\in\mathcal{L}^*_{x}}{\quad} p_i \underset{j\neq i}{\quad} p_j\ \ d\Lambda^*_{x_i} - \underset{p\in\mathcal{L}^*_{x}}{\quad} p_i\, d\Lambda^*_{x_i} \underset{p\in\mathcal{L}^*_{x}}{\quad}\underset{j\neq i}{\quad} p_j\ \ d\Lambda^*_{x_i}$$

$p \in [0,1]^N$

$\delta_i(x) > 0$ (< 0) $x \in Y_i(x_i)$. H P 1 (i, x), $\Psi_i(x_i) > 0$ (< 0) $\delta_i(x) > 0$ (< 0) $\Lambda^*_{x_i}$ - i' . I (6) $\Lambda^*_{x_i}$ i' , $\Psi_i(x_i) = 0$. . . .

T $\Lambda^*_{x_i}$ i' - A 2. F , $\delta_i(x) \neq 0$ $(u_j, \delta_j, F_{\varepsilon_j|X})$ $j \neq i$ i. T - i x $(\delta_i(x) = 0)$.[8] P ( ) P 2 - . T (i, x) .

EXAMPLE 2 N A i, x: C 2- -2 1 2 X $(\tilde{X}_0, \tilde{X}_1, \tilde{X}_2)$, $\tilde{X}_0$ - $\tilde{X}_1$ $\tilde{X}_2$ - - 1 2, . S $\delta_i(x) \neq 0$ i, x, A 1 2 $X_1 \equiv (\tilde{X}_0, \tilde{X}_1)$ $X_2 \equiv (\tilde{X}_0, \tilde{X}_2)$. A i i x Λ$_x^*$ i' BNE x.

4. TESTING MULTIPLE BNE AND INFERRING INTERACTION SIGNS

W , (., B D (2007, . 58)). H , BNE - . T , - , (-) δ(·) . W , . N , , . H , .

A , (-

I S 3 $\Delta_i(x) \neq 0$ i -
x. I S M ,
W (7) x. W
$\Delta_i(x)$
( ) $f_i(x; D_{-i})$.

A DGP
(1)
$u, \delta,$ $F_{\varepsilon|x}$.[11] I BNE -
, N (7)
. W
- S 4.1 .

F , $N = 2$, BNE x -
. I , -

11

i x:

$$H_i^0 : \square_i(x) = 0$$
$$H_i^1 : \square_i(x) \neq 0$$

G $i \leq N$, *f* (FWE), , . T ,

$$\text{FWE}_P = P_P\{ \quad H_i^0 : \square_i(x) = 0 \quad i \in \mathbf{I}_0(P)\}$$

P DGP $\mathbf{I}_0(P) \subset \{1 \ldots N\}$ i P. A - - FWE_P α $_{G \to +\infty}$ $\text{FWE}_P \leq \alpha$ P. W Ω_X x . S x - $X = x$. W Ω_X , X - - (. .,). S , , Ω_X . N [illegible] $\hat{\boldsymbol{\mu}}_G$ [illegible] μ. B[illegible], $G^{1/2}(\hat{\boldsymbol{\mu}}_G(\{x\}) - \boldsymbol{\mu}(\{x\})) \xrightarrow{d} \mathcal{N}(\mathbf{0}_{\tilde{N}}, \boldsymbol{\Sigma}(\{x\}))$ [illegible] $G \to \infty$, [illegible] $\mathbf{0}_{\tilde{N}}$ [illegible] $\tilde{N}$ [illegible] Σ [illegible]. D[illegible] $\mathbf{T}_G(\{x\})$ [illegible] N [illegible] i [illegible]

$$T_{G,i}(\{x\}) \equiv \sum_{j \neq i} \left[\frac{\hat{\mu}_{ij}(\{x\})}{\hat{\mu}_0(\{x\})} - \frac{\hat{\mu}_i(\{x\})\hat{\mu}_j(\{x\})}{(\hat{\mu}_0(\{x\}))^2} \right].$$

B[illegible],

$$G^{1/2}\left(\mathbf{T}_G(\{x\}) - \boldsymbol{\Delta}(x)\right) \xrightarrow{d} N\left(\mathbf{0}_N, \mathbf{V}(\{x\})\boldsymbol{\Sigma}(\{x\})\mathbf{V}(\{x\})'\right)$$

[illegible] $G \to \infty$,

[illegible] $\boldsymbol{\Delta}(x) \equiv (\Delta_i(x))_{i=1}^N$. T[illegible] J[illegible] $\mathbf{V}(\{x\})$ [illegible] N-[illegible]-$\tilde{N}$ [illegible], [illegible] i [illegible] $V_i(\{x\})$ [illegible] ([illegible] $\mu_{(m)}(\{x\}), V_{i,(m)}(\{x\})$ [illegible] m [illegible] $\tilde{N}$ [illegible] $\boldsymbol{\mu}(\{x\})$ [illegible] $V_i(\{x\})$, [illegible], [illegible] $j, k \neq i$):

$\mu_{(m)}(\{x\})$	$V_{i,(m)}(\{x\})$
$\mu_0(\{x\})$:	$\sum_{j \neq i}\left(-\frac{\mu_{ij}(\{x\})}{\mu_0(\{x\})^2} + \frac{2\mu_i(\{x\})\mu_j(\{x\})}{\mu_0(\{x\})^3}\right)$
$\mu_i(\{x\})$:	$-\sum_{j \neq i} \frac{\mu_j(\{x\})}{\mu_0(\{x\})^2}$
$\mu_j(\{x\})$:	$-\frac{\mu_i(\{x\})}{\mu_0(\{x\})^2}$
$\mu_{ij}(\{x\})$ [illegible] $\mu_{ji}(\{x\})$:	$\frac{1}{\mu_0(\{x\})}$
$\mu_{jk}(\{x\})$:	0

W[illegible] $\boldsymbol{\Sigma}(\{x\}), \mathbf{V}(\{x\})$ [illegible] $\mu_0(\{x\}), \mu_I(\{x\})$ [illegible]. F[illegible], [illegible] $(\{x\})$ [illegible].

W[illegible] B[illegible] H[illegible]'[illegible]. B[illegible] p-[illegible] ([illegible] i). [illegible]. W[illegible] p-[illegible] $\hat{p}_{G,i}$. T[illegible] B[illegible] α [illegible] i [illegible] $\hat{p}_{G,i} \leq \alpha/N$. T[illegible] H[illegible]'[illegible], [illegible] B[illegible], [illegible]. (F[illegible], [illegible] x.) T[illegible] H[illegible]'[illegible] p-[illegible]: $\hat{p}_{G,(1)} \leq \hat{p}_{G,(2)} \leq \cdots \leq \hat{p}_{G,(N)}$. L[illegible] $H_{j_k}^0$:

.[15] W $\hat{c}_k$ S -

S ’ ,

$$\left(\hat{V}_i(Y_i(x))\hat{\boldsymbol{\Sigma}}(Y_i(x))\hat{V}_i(Y_i(x))'/G\right)^{-1/2}$$

$$\times \left(T_{G,i}(Y_i(x)) - \Psi_i(x)\right) \xrightarrow{d} \mathcal{N}(0,1) \quad G \to \infty$$

$\hat{V}_i(Y_i(x))$ $\hat{\boldsymbol{\Sigma}}(Y_i(x))$ $V_i(Y_i(x))$ $\boldsymbol{\Sigma}(Y_i(x))$, S 4.1. T $\delta_i(x)$

F (λ, G), ’

$$D_{i,g} = 1 \quad u_i - W_g \sum_{j\neq i} p_j^1 - (1 - W_g) \sum_{j\neq i} p_j^2 - \varepsilon_{i,g} \geq 0$$

, $g \leq G$, W_g B λ, $\varepsilon_{i,g}$ $\mathcal{N}(0, 1, 0.25^2)$ p^l’ B N . F (λ, G), $S = 1000$. F , - - S 4.2 - . W - $\hat{c}_k$ S 4.2: ( ) T_G, ( ) , ( ) (A 3.2 4.2 R W (2005)). F , - ( ) ( ) ( ) (B). W $B = 1000, 2000$. I T I, (.., H_0 $i = 1$) $S = 1000$ RP1, RP2, RP3, RP .

[redacted]	i = 3
[redacted]	[0.000, 1.000]
	[0.000, 1.000]
	[0.000,1.000]
	[0.000, 1.000]
	[0.000, 1.000]
	[0.000, 1.000]

[redacted]

TABLE IV

FINITE SAMPLE PERFORMANCE: TEST OF SIGNS OF INTERACTION EFFECTS

	$G = 5000$			$G = 10{,}000$		
	$\delta = 0.8$	$\delta = 0.9$	$\delta = 1.0$	$\delta = 0.8$	$\delta = 0.9$	$\delta = 1.0$
$X_1 = -1$	[0.000, 0.469]	[0.001, 0.628]	[0.000, 0.854]	[0.000, 0.717]	[0.000, 0.890]	[0.000, 0.986]
$X_2 = -1/2$	[0.003, 0.359]	[0.000, 0.520]	[0.000, 0.714]	[0.000, 0.577]	[0.000, 0.790]	[0.000, 0.925]
$X_3 = -1$	[0.000, 0.483]	[0.000, 0.643]	[0.000, 0.834]	[0.000, 0.702]	[0.000, 0.888]	[0.000, 0.986]
$X_1 = 2$	[0.323, 0.004]	[0.459, 0.000]	[0.667, 0.000]	[0.484, 0.000]	[0.736, 0.000]	[0.910, 0.000]
$X_2 = 3/2$	[0.400, 0.000]	[0.617, 0.000]	[0.817, 0.000]	[0.665, 0.000]	[0.867, 0.000]	[0.979, 0.000]
$X_3 = 3$	[0.300, 0.004]	[0.496, 0.000]	[0.735, 0.000]	[0.545, 0.000]	[0.764, 0.000]	[0.930, 0.000]

N ... $S = 1000$. T ... $[q_+, q_-]$: q_+ ... H_0 ... H_+; q_- ... H_0 ... H_-.

... 1 ... x. A ..., $\Lambda^*_{x_i}(p) = 1/4$ ... $p \in \{(0.3233, 0.5603, 0.3233), (0.2523, 0.5288, 0.7098), (0.2998, 0.7013, 0.2998), (0.2101, 0.7262, 0.7231)\}$, ... T ... 1 ... , ... 1. A ... X_1

1, strateic, eiffcpts a srat-dependent ... for player 1, srates the εrstra

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TABLE VI

MULTIPLICITY TESTS (X = HOUR OF DAY)

		:55 :55	G
A	W	33[illegible]32*	26,152
	RW	$T_1^{\dagger} > T_3^{\dagger} > T_2^{\dagger} > 0$	
N 1	W	3[illegible]86	6534
	RW	$T_3 > T$	

TABLE VII

:55 MIN VS. NOT :55 MIN (X = HOUR OF DAY, MARKET SIZE)

M[illegible] S[illegible] ([illegible])		H[illegible] D[illegible]			
		N[illegible] 1[illegible]	4 5[illegible]	5 6[illegible]	9 10[illegible]
1	W[illegible]	0.77	4.94	3.22	2.27
	RW	$T_3 > T_2 > 0 > T_1$	$T_2 > T_1 > T_3 > 0$	$T_2 > T_1 > T_3 > 0$	$T_1 > T_3 > T_2 > 0$
	G	2201	2201	2200	2199
2	W[illegible]	0.73	3.87	1.97	2.48
	RW	$T_2 > T_3 > 0 > T_1$	$T_3 > 0 > T_1 > T_2$	$T_2 > T_1 > T_3 > 0$	$T_2 > T_1 > T_3 > 0$
	G	2157	2220	2159	2153
3	W[illegible]	4.96	19.06*	26.07*	2.92
	RW	$T_2 > T_3 > T_1 > 0$	$T_3^\dagger > T_2^\dagger > T_1^\dagger > 0$	$T_1^\dagger > T_3^\dagger > T_2^\dagger > 0$	$T_1 > T_3 > 0 > T_2$
	G	2176	2141	2177	2168

T[illegible] (*) [illegible] W[illegible] [illegible] 5%. T[illegible] (†) [illegible] 5% FWE [illegible] R[illegible] W[illegible] (2005) (RW). T_k [illegible] ([illegible]) k. T[illegible] 1000.

[illegible] 4 5 [illegible] 5 6 [illegible].

7. CONCLUSION

I[illegible]. I[illegible]. I[illegible], [illegible]. E[illegible], [illegible] ([illegible]).

W[illegible], [illegible]. W[illegible], [illegible]. W[illegible]. A[illegible].

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