

Appendix A

Zero-Current Wavefunctions

One interesting aspect of the detector discussed in Section 3.3.2, is that while it can be used for wave-packets arriving from the left or the right, it will not always be triggered if the wavefunction is a coherent superposition of right and left moving modes. Consider for example, the superposition

$$\psi(x) = Ae^{ikx} + Ae^{-ikx}. \quad (\text{A.216})$$

One can easily verify that the current

$$j(x, t) = -i\frac{1}{2m} \left[\psi^*(x, t) \frac{\partial \psi(x, t)}{\partial x} - \frac{\partial \psi^*(x, t)}{\partial x} \psi(x, t) \right] \quad (\text{A.217})$$

is zero in this case. $|\psi(0, t)|^2$ is non-zero, although the state is not normalizable. As in eq. (3.63) this state evolves into

$$\langle x | \psi \rangle | \uparrow_z \rangle \rightarrow \frac{A}{\sqrt{2}} \left[(e^{ikx} + e^{-ikx}) | \uparrow_x \rangle + (e^{ikx} - e^{-ikx}) | \downarrow_x \rangle \right] \quad (\text{A.218})$$

W

Appendix B

Gaussian Wave Packet and Clocks

Using the simple model of Section 3.3.1 (3.44), we now calculate the probability distribution of a clock which measures the time-of-arrival of a Gaussian wave packet. We will perform the calculation in the limits when the clock is extremely accurate and extremely inaccurate. The wave function of the clock and particle is given by (3.52) and the distributions are both Gaussians given by (3.53). In the inaccurate limit, when $p_o \ll k$, $A_T \sim 1$. W

Since $\Delta y k \gg 1$, for a wave packet peaked around k_o we can approximate the argument of the first exponential by $\frac{-\Delta y^2 k_o^2}{2m^2}(k - k')^2$. This allows us to integrate over k and k'

$$\rho(y, y)_{>0} \simeq \frac{1}{\sqrt{2\pi\gamma(y)}} e^{-\frac{(y-t_c)^2}{2\gamma(y)}} \quad (\text{B.222})$$

here the width is $\gamma(y) = \Delta y^2 + (\frac{m\Delta x}{k_o})^2 + (\frac{y}{2k_o\Delta x})^2$.

As expected, the distribution is centered around the classical time-of-arrival $t_c = x_o m/k_o$. The spread in y has a term due to the initial width Δy in clock position y . The second and third term in $\gamma(y)$ is due to the kinematic spread in the time-of-arrival $1/dE$

